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Akhlesh Lakhtakia[a]; Werner S. Weiglhofer[b]
[a] Department of Engineering Science and Mechanics, The Pennsylvania State University, University Park, Pennsylvania, U.S.A. [b] Department of Mathematics, University of Glasgow, Glasgow, Scotland

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Axial propagation in a magnetic-dielectric cholesteric medium

by AKHLESH LAKHTAKIA*
Department of Engineering Science and Mechanics, The Pennsylvania State University, University Park, Pennsylvania 16802-1401, U.S.A.

and WERNER S. WEIGLHOFER
Department of Mathematics, University of Glasgow, Glasgow G12 8QW, Scotland

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The existence of an exact solution for axial propagation in a dielectric cholesteric medium has been often remarked upon. Here, we obtain the exact solution for axial propagation in a magnetic-dielectric cholesteric medium, using the 4×4 matrix method of optics. Our solution procedure also enables us to identify the specific features that make axial propagation in cholesteric media remarkable in having an analytical solution.

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1. Introduction

Whereas planewave propagation in uniaxial dielectric materials has been intensively studied for well over three half-centuries [1–4], general exact analytical solutions for planewave propagation in cholesteric liquid crystals still remain elusive. This may be because cholesteric liquid crystals are periodically inhomogeneous uniaxial dielectric materials, and exact solutions for linear differential equations with periodic coefficients are rather rare [5–7].

It is possible to make the piecewise constant approximation [8, 9] of replacing a cholesteric slab by a multilayered uniaxial dielectric slab, and a simplified version of the resulting algorithm has been numerically implemented [10, 11]. Perturbative [12], long wavelength or quasistatic [9, 13], and the geometrical optics [14, 15] approximations have also been used for a cholesteric medium of infinite extent, and note may also be taken of the numerical implementation [16] of the Floquet–Lyapunov theorem [6]. However, as Belyakov [5] has so eloquently put it, an exact, closed form solution has been found only for propagation parallel to the helical axis [17, 18]. This assessment holds true in spite of some analytical advances reported in the past 10 years: Oldano *et al.* [19] have expanded the electric field in terms of its Floquet–Bloch planewave spectrum but their numerical procedure has to be carefully checked for convergence; Peterson [20] has utilized a cylindrical wave spectrum instead, but his solution involves a matrix continued fraction and also requires the numerical solution of a high order polynomial equation; Kapshai *et al.* [21] are closer to a closed form solution, but have only been able to give approximate expressions for the eigenfields.

The existence of an exact solution for axial propagation in a cholesteric medium is so remarkable that it is worth quoting Belyakov [5]: 'The obtained exact solution is

* Author for correspondence.

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simple and is the only example of a simple exact analytical solution of the Maxwell equations for periodic media. No other simple exact solution is known—not even for periodic structures more simple than cholesterics...'. Belyakov's comments naturally lead to a question: What are the essential features of this problem? The mode of presenting its solution in textbooks (for instance, Belyakov [5], de Gennes [22] or Chandrasekhar [23]) does not illuminate the uniqueness of its existence. This may be because, conventionally, resort is made to second order differential equations that often obscure some special features of the first order differential equations of Maxwell. True, Berreman and Scheffer [10, 11] did use the first order equations, but only together with the piecewise constant approximation.

A general material can have dielectric as well as magnetic properties [24, 25], so whatever has been written above for *dielectric cholesteric* media applies in full force to *magnetic cholesteric* media [26, 27] as well. A magnetic cholesteric (ferrocholesteric) medium can be made by suspending acicular magnetic particles in a carrier liquid crystal and then cooling under/without a magnetic field pulse to establish long range orientational order for the magnetic needles [26, 28]. Continual advances being reported on the fabrication of these types of materials prompt the second question: Is the exact, closed form solution for axial propagation in a dielectric cholesteric medium a *reduction* of another exact, closed form solution for axial propagation in a *magnetic-dielectric cholesteric* medium?

The present communication is being made to answer these two questions. Using the famous 4×4 matrix method of optics [7, 9, 10], we obtain the exact solution for axial propagation in a magnetic-dielectric cholesteric medium. In doing so, we find out the answers to the two questions hitherto posed.

At the risk of appearing repetitive, let us pause to enumerate the reasons for the existence of this paper. First, the publications of de Vries [17] and Kats [18] for axial propagation in a dielectric cholesteric medium were the first in which the time-harmonic Maxwell equations were solved in a periodically inhomogeneous medium; any extension thereof is of analytical value, and even more so when the extension has not been reported elsewhere. Second, we have very clearly obtained in § 4 the reasons why the deVries–Kats approach works, which has not been done elsewhere to our knowledge; that fact endows our paper with pedagogical value. Third, though the anisotropy of the diamagnetic susceptibility of liquid crystals is usually very small, this can be enhanced in the manner proposed by Brochard and de Gennes [26] by suspending acicular magnetic particles in dielectric cholesterics. Such materials can nowadays be made, albeit with considerable difficulty [27].

2. Field equations

Frequency-domain electromagnetic fields, in the magnetic-dielectric cholesteric material we are handling here, obey the constitutive relations

$$\mathbf{D}(\mathbf{r}) = \mathfrak{e}(z)\cdot\mathbf{E}(\mathbf{r}), \tag{1a}$$

$$\mathbf{B}(\mathbf{r}) = \mathfrak{m}(z)\cdot\mathbf{H}(\mathbf{r}), \tag{1b}$$

where $\mathfrak{e}(z)$ is the permittivity dyadic and $\mathfrak{m}(z)$ is the permeability dyadic. The permittivity dyadic is specified as

$$\mathfrak{e}(z) = \varepsilon_0[\varepsilon_a\mathfrak{I} + (\varepsilon_b - \varepsilon_a)\mathbf{u}_e(z)\mathbf{u}_e(z) + (\varepsilon_c - \varepsilon_a)\mathbf{u}_z\mathbf{u}_z], \tag{2a}$$

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where $\mathfrak{I}=\mathbf{u}_x\mathbf{u}_x+\mathbf{u}_y\mathbf{u}_y+\mathbf{u}_z\mathbf{u}_z$ is the identity dyadic and $(\mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_z)$ is the triad of cartesian unit vectors; $\varepsilon_0=8{\cdot}854\times 10^{-12}\,\mathrm{F\,m^{-1}}$ is the permittivity of free space, and $(\varepsilon_a, \varepsilon_b, \varepsilon_c)$ is the triad of relative permittivities; while in specifying the dielectric director

$$\mathbf{u}_e(z)=\mathbf{u}_x\cos\phi_e(z)+\mathbf{u}_y\sin\phi_e(z) \tag{2 b}$$

that spins around the z axis, the axis of spirality in the material, we make no particular assumptions regarding the dependence of the angle $\phi_e(z)$ on z at this stage. The permeability dyadic is set up as

$$\mathfrak{m}(z)=\mu_0[\mu_a\mathfrak{I}+(\mu_b-\mu_a)\mathbf{u}_m(z)\mathbf{u}_m(z)+(\mu_c-\mu_a)\mathbf{u}_z\mathbf{u}_z], \tag{3 a}$$

where $\mu_0=4\pi\times 10^{-7}\,\mathrm{H\,m^{-1}}$ is the permeability of free space, (μ_a, μ_b, μ_c) is the triad of relative permeabilities, and the angle $\phi_m(z)$ of the magnetic director

$$\mathbf{u}_m(z)=\mathbf{u}_x\cos\phi_m(z)+\mathbf{u}_y\sin\phi_m(z) \tag{3 b}$$

has an arbitrary dependence on z at this juncture.

We are interested solely in axial propagation. Therefore, $\partial/\partial x=\partial/\partial y\equiv 0$ and the field vectors depend on the z coordinate only; hence, $\mathbf{E}(\mathbf{r})\equiv\mathbf{E}(z)$, etc. Substitution of the preceding equations in the time-harmonic Maxwell curl equations, $\nabla\times\mathbf{E}=i\omega\mathbf{B}$ and $\nabla\times\mathbf{H}=-i\omega\mathbf{D}$, yields us the two algebraic relations $0=\mathbf{u}_z\cdot\mathbf{B}(z)$ and $0=\mathbf{u}_z\cdot\mathbf{D}(z)$. This implies that the $\mathbf{B}$ and the $\mathbf{D}$ fields are purely transverse for axial propagation, as also are the $\mathbf{E}$ and the $\mathbf{H}$ fields.

In addition to the two algebraic equations, we also get four first order differential equations that can be put in matrix form as

$$(d/dz)[f(z)]=[A(z)][f(z)], \tag{4 a}$$

where $[f(z)]$ is the column vector

$$[f(z)]=\begin{bmatrix} E_x(z)\\ E_y(z)\\ H_x(z)\\ H_y(z)\end{bmatrix}, \tag{4 b}$$

and $[A(z)]$ is the 4×4 matrix

$$[A(z)]=i\omega$$

$$\begin{bmatrix} 0 & 0 & \mu_0\mu_-\sin 2\phi_m & \mu_0\mu_+-\mu_0\mu_-\cos 2\phi_m\\ 0 & 0 & -\mu_0\mu_+-\mu_0\mu_-\cos 2\phi_m & -\mu_0\mu_-\sin 2\phi_m\\ -\varepsilon_0\varepsilon_-\sin 2\phi_e & -\varepsilon_0\varepsilon_++\varepsilon_0\varepsilon_-\cos 2\phi_e & 0 & 0\\ \varepsilon_0\varepsilon_++\varepsilon_0\varepsilon_-\cos 2\phi_e & \varepsilon_0\varepsilon_-\sin 2\phi_e & 0 & 0\end{bmatrix}, \tag{4 c}$$

with

$$\varepsilon_\pm=(\varepsilon_b\pm\varepsilon_a)/2, \tag{5 a}$$

$$\mu_\pm=(\mu_b\pm\mu_a)/2. \tag{5 b}$$

The solution of the matrix differential equation (4 a) is what we are after.

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3. Field transformation

It is conventional to think of left- and right-handed circularly polarized fields for axial propagation in a dielectric cholesteric [20, 22]). In addition, we know that $\mathfrak{e}(z)$ and $\mathfrak{m}(z)$ are periodic functions of z; indeed, the factorizations

$$\mathfrak{e}(z)=\{\exp[\phi_e(z)\mathbf{u}_e(z)\times\mathfrak{I}]\}\cdot\mathfrak{e}(0)\cdot\{\exp(-\phi_e(z)\mathbf{u}_e(z)\times\mathfrak{I}]\}, \tag{6a}$$

$$\mathfrak{m}(z)=\{\exp[\phi_m(z)\mathbf{u}_m(z)\times\mathfrak{I}]\}\cdot\mathfrak{m}(0)\cdot\{\exp[-\phi_m(z)\mathbf{u}_m(z)\times\mathfrak{I}]\} \tag{6b}$$

clearly show the rotational nature of the material inhomogeneity. Therefore, we make the mapping

$$[f(z)]=[P][F(z)][\gamma(z)], \tag{7a}$$

where the constant matrix

$$[P]=2^{-1/2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ i & -i & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & i & -i \end{bmatrix} \tag{7b}$$

converts the electromagnetic fields from the linearly to the circularly polarized bases, while the matrix

$$[F(z)]=\begin{bmatrix} \exp[-i\phi_e(z)] & 0 & 0 & 0 \\ 0 & \exp[-i\phi_e(z)] & 0 & 0 \\ 0 & 0 & \exp[-i\phi_m(z)] & 0 \\ 0 & 0 & 0 & \exp[i\phi_e(z)] \end{bmatrix} \tag{7c}$$

is purely diagonal.

It follows from (7 a) that

$$(d/dz)[f(z)]=[P]\{(d/dz)[F(z)]\}[\gamma(z)]+[P][F(z)]\{(d/dz)[\gamma(z)]\}. \tag{8}$$

Hence, the matrix differential equation (4 a) can be transformed to

$$(d/dz)[\gamma(z)]=[C(z)][\gamma(z)], \tag{9a}$$

where

$$[C(z)]=[F(z)]^{-1}[P]^{-1}[A(z)][P][F(z)]-i[Q(z)], \tag{9b}$$

with

$$[Q(z)]=\begin{bmatrix} -d\phi_e/dz & 0 & 0 & 0 \\ 0 & d\phi_e/dz & 0 & 0 \\ 0 & 0 & -d\phi_m/dz & 0 \\ 0 & 0 & 0 & d\phi_m/dz \end{bmatrix}. \tag{9c}$$

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4. A conditional solution

The transformed field equation (9 *a*) involves the matrix $[C(z)]$ of (9 *b*). The structure of $[C(z)]$ is the key of the present investigation. It turns out that

$$[C(z)]=\begin{bmatrix} i d\phi_e/dz & 0 & -\omega\mu_0\mu_+\exp[-i\delta(z)] & -\omega\mu_0\mu_-\exp[-i\delta(z)] \\ 0 & -i d\phi_e/dz & \omega\mu_0\mu_-\exp[i\delta(z)] & \omega\mu_0\mu_+\exp[i\delta(z)] \\ \omega\varepsilon_0\varepsilon_+\exp[i\delta(z)] & \omega\varepsilon_0\varepsilon_-\exp[i\delta(z)] & i d\phi_m/dz & 0 \\ -\omega\varepsilon_0\varepsilon_-\exp[-i\delta(z)] & -\omega\varepsilon_0\varepsilon_+\exp[-i\delta(z)] & 0 & -i d\phi_m/dz \end{bmatrix} \tag{10}$$

with

$$\delta(z)=\phi_m(z)-\phi_e(z). \tag{11}$$

If $[C(z)]$ can be made independent of z, the solution of (9 *a*) is quite straightforward. To make $[C(z)]$ constant with respect to z introduces the following requirements:

$$\text{(i)}\quad d\phi_e/dz=b_e, \tag{12 a}$$

$$\text{(ii)}\quad d\phi_m/dz=b_m, \tag{12 b}$$

$$\text{(iii)}\quad \delta(z)=\delta_0, \tag{12 c}$$

where b_e, b_m and δ_0 are constants. If these three conditions are met, (9 *a*) reduces to the matrix differential equation

$$(d/dz)[\gamma(z)]=[C_0][\gamma(z)] \tag{13 a}$$

that has the constant matrix

$$[C_0]=\begin{bmatrix} ib_e & 0 & -\omega\mu_0\mu_+\exp(-i\delta_0) & -\omega\mu_0\mu_-\exp(-i\delta_0) \\ 0 & -ib_e & \omega\mu_0\mu_-\exp(i\delta_0) & \omega\mu_0\mu_+\exp(i\delta_0) \\ \omega\varepsilon_0\varepsilon_+\exp(i\delta_0) & \omega\varepsilon_0\varepsilon_-\exp(i\delta_0) & ib_m & 0 \\ -\omega\varepsilon_0\varepsilon_-\exp(-i\delta_0) & -\omega\varepsilon_0\varepsilon_+\exp(-i\delta_0) & 0 & -ib_m \end{bmatrix} \tag{13 b}$$

for its kernel. The solution of (13 *a*) can then be obtained as [29]

$$[\gamma(z)]=\exp\{[C_0](z-z')\}[\gamma(z')], \tag{13 c}$$

whence the simple result

$$[f(z)]=[P][F(z)]\exp\{[C_0](z-z')\}[F(z')]^{-1}[P]^{-1}[f(z')]. \tag{13 d}$$

The matrix $\exp\{[C_0](z-z')\}$ can be simplified using the eigenvalues and the eigenvectors of $[C_0]$. We note that the eigenvalue equation for the $[C_0]$ of (13 *b*) is a quartic and is best handled numerically, even though the cubic term is missing therefrom.

The above procedure makes it clear why an analytical solution for axial propagation in a *dielectric cholesteric* medium can be found. If 2Ω is the pitch of the cholesteric, we have $\phi_e(z)=\phi_m(z)=\pi z/\Omega$ and $\mu_b=\mu_a$. Ergo, $b_e=b_m=\pi/\Omega$ and $\delta_0=0$; and all three conditions (12 *a, b, c*) are fulfilled. The matrix $[C(z)]$ thus becomes independent of z and an exact solution of the form (13 *b*) becomes attainable.

5. Magnetic-dielectric cholesteric medium

We may now proceed with the application of the results of the previous section for axial propagation in the magnetic-dielectric cholesteric medium described in § 2. We use

$$\phi_e(z)=\pi z/\Omega, \tag{14a}$$

$$\phi_m(z)=\pi z/\Omega+\delta_0, \tag{14b}$$

to obtain

$$[C_0]=\begin{bmatrix} i\pi/\Omega & 0 & -\omega\mu_0\mu_+\exp(-i\delta_0) & -\omega\mu_0\mu_-\exp(-i\delta_0) \\ 0 & -i\pi/\Omega & \omega\mu_0\mu_-\exp(i\delta_0) & \omega\mu_0\mu_+\exp(i\delta_0) \\ \omega\varepsilon_0\varepsilon_+\exp(i\delta_0) & \omega\varepsilon_0\varepsilon_-\exp(i\delta_0) & i\pi/\Omega & 0 \\ -\omega\varepsilon_0\varepsilon_-\exp(-i\delta_0) & -\omega\varepsilon_0\varepsilon_+\exp(-i\delta_0) & 0 & -i\pi/\Omega \end{bmatrix}. \tag{14c}$$

This matrix is diagonalizable in the form

$$[C_0]=[T][G][T]^{-1}, \tag{15a}$$

where the diagonal matrix

$$[G]=\begin{bmatrix} g_1 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 \\ 0 & 0 & g_3 & 0 \\ 0 & 0 & 0 & g_4 \end{bmatrix} \tag{15b}$$

contains the four eigenvalues g_n, $n=1$–4, of $[C_0]$. The successive columns of the matrix

$$[T]=\begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{bmatrix} \tag{15c}$$

are the corresponding eigenvectors of $[C_0]$. The quantities $-ig_n$ can be interpreted as wavenumbers.

The eigenvalues of the matrix $[C_0]$ of (14 *c*) are obtained by setting the determinant of the matrix $([C_0]-g[I])$ equal to zero, where $[I]$ is the identity matrix. That process yields the biquadratic equation

$$(g^2+\pi^2/\Omega^2)^2+\alpha(g^2+\pi^2/\Omega^2)+\beta=0, \tag{16a}$$

where

$$\alpha=2k_0^2(\mu_+\varepsilon_+-\mu_-\varepsilon_-\cos 2\delta_0), \tag{16b}$$

$$\beta=k_0^4(\mu_+^2-\mu_-^2)(\varepsilon_+^2-\varepsilon_-^2)-4k_0^2(\pi/\Omega)^2\mu_+\varepsilon_+, \tag{16c}$$

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and $k_0=\omega\sqrt{(\mu_0\varepsilon_0)}$ is the free space wavenumber. It follows from (16 *a*) that the four eigenvalues are

$$g_1=\sqrt{(\lambda_- -\pi^2/\Omega^2)}, \quad g_2=-g_1, \tag{17 a, b}$$

$$g_3=\sqrt{(\lambda_+ -\pi^2/\Omega^2)}, \quad g_4=-g_3, \tag{17 c, d}$$

where

$$\lambda_\pm=-(\alpha/2)\pm(1/2)\sqrt{(\alpha^2-4\beta)}. \tag{17 e}$$

That we obtain $g_3=-g_1$ and $g_4=-g_2$ is reassuring, because the magnetic-dielectric cholesteric material under investigation is reciprocal as its $\mathfrak{e}(z)$ and $\mathfrak{m}(z)$ are symmetric dyadics [30].

The elements of the *n*th eigenvector are, quite simply, given in an unnormalized form as

$$t_{n1}=-\omega\mu_0[\mu_-(g_n^2+\pi^2/\Omega^2)+\exp(2i\delta_0)k_0^2\varepsilon_-(\mu_+^2-\mu_-^2)], \tag{18 a}$$

$$t_{n2}=\omega\mu_0\exp(2i\delta_0)[\mu_+(g_n-i\pi/\Omega)^2+k_0^2\varepsilon_+(\mu_+^2-\mu_-^2)], \tag{18 b}$$

$$t_{n3}=-k_0^2\exp(i\delta_0)(g_n+i\pi/\Omega)\mu_-\varepsilon_+ +k_0^2\exp(3i\delta_0)(g_n-i\pi/\Omega)\mu_+\varepsilon_-, \tag{18 c}$$

$$t_{n4}=\exp(i\delta_0)[(g_n^2+\pi^2/\Omega^2)(g_n-i\pi/\Omega)+k_0^2(g_n+i\pi/\Omega)\mu_+\varepsilon_+] \\ -k_0^2\exp(3i\delta_0)(g_n-i\pi/\Omega)\mu_-\varepsilon_-. \tag{18 d}$$

As

$$\exp\{[C_0](z-z')\}=[T]\exp\{[G](z-z')\}[T]^{-1} \tag{19}$$

in view of (15 *a*), axial progagation takes place as described by the relation

$$[f(z)]=[X(z)]\exp\{[G](z-z')\}[X(z')]^{-1}[f(z')], \tag{20}$$

wherein we have introduced the matrix

$$[X(z)]=[P][F(z)][T]. \tag{21}$$

Equivalently, after setting $z'=0$ without loss of generality, we can cast the solution in the form

$$[f(z)]=[X(z)]\exp\{[G]z\}[f_0] \tag{22 a}$$

where the coefficient vector

$$[f_0]=[X(0)]^{-1}[f(0)]. \tag{22 b}$$

The solution given as (22 *a*) is not in a form compatible with the Floquet–Lyapunov theorem [6, 9, 12], but a slightly modified form is. To obtain the latter form, let us define the matrix

$$[Y(z)]=[X(z)][F(z)]^{-1} \tag{23}$$

and transform (20) to

$$[f(z)]=[Y(z)]\exp\{i[K](z-z')\}[Y(z')]^{-1}[f(z')], \tag{24}$$

where

$$[K]=\begin{bmatrix} -ig_1-\pi/\Omega & 0 & 0 & 0 \\ 0 & -ig_2+\pi/\Omega & 0 & 0 \\ 0 & 0 & -ig_3-\pi/\Omega & 0 \\ 0 & 0 & 0 & -ig_4+\pi/\Omega \end{bmatrix}. \tag{25}$$

Then, setting $z'=0$ again, we can recast (24) into

$$[f(z)]=[Y(z)]\exp\{i[K]z\}[f_{00}], \tag{26 a}$$

where the coefficient vector

$$[f_{00}]=[Y(0)]^{-1}[f(0)]. \tag{26 b}$$

Because $[Y(z+\Omega)]=[Y(z)]$ and $[K]$ is independent of z, the solution (26 a) is in the form required by the Floquet–Lyapunov theorem.

The spirality of the medium vanishes in the limit $\Omega\rightarrow\infty$, hence the inhomogeneity also. This provides us with a means to test the derivations made in this section, for which purpose we looked at the gyroelectromagnetic uniaxial medium [25, 31] by taking the limit $\Omega\rightarrow\infty$ along with $\delta_0=0$. The medium under investigation here thereby became homogeneous, and we found that the solution (21 a) reduced to published results [32] for axial propagation in the gyroelectromagnetic uniaxial medium.

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6. Final remarks

We have thus obtained an analytical solution for axial propagation in a magnetic-dielectric cholesteric or ferrocholesteric medium. This medium is more general than the usual dielectric cholesteric medium. Therefore, we have shown that the exact, closed form solution for axial propagation in a dielectric cholesteric medium is a reduction of another exact, closed form solution for axial propagation in a magnetic-dielectric cholesteric medium.

The features that allowed this exact solution are the following:

(i) The electromagnetic fields as well as the permittivity and the permeability dyadics vary only with the propagation direction.

(ii) The inhomogeneity is purely rotational, as demonstrated by (6 a, b), and transverse to the propagation direction.

(iii) The effective anisotropy of the medium is purely transverse to the propagation direction, the (non-zero) values of $\mathbf{u}_z\cdot\mathrm{e}(z)\cdot\mathbf{u}_z(=\varepsilon_c\varepsilon_0)$ and $\mathbf{u}_z\cdot\mathrm{m}(z)\cdot\mathbf{u}_z(\mu_c\mu_0)$ being inconsequential for the present problem.

(iv) The angles $\phi_e(z)$ and $\phi_m(z)$ vary linearly with the propagation direction, the difference between the two being a constant.

Our original question, as to what the special features are that permit an analytical solution, has therefore been answered; and we note that the answer is in a more general setting than the question. To conclude, we add that the understanding obtained here by us has been of assistance in obtaining exact and simple solutions for propagation in general helicoidal media [33].

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